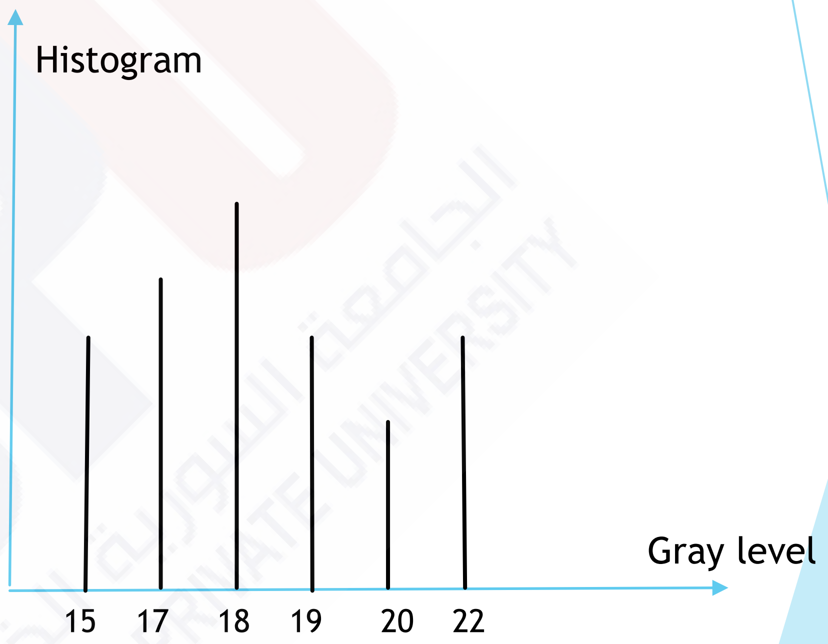
What is a Histogram of an image ?

The histogram of a digital image with gray levels in the range $[0, L-1]$ is a discrete function $h(r_k) = n_k$, where r_k is the kth gray level and n_k is the number of pixels in the image having gray level r_k. It is common practice to normalize a histogram by dividing each of its values by the total number of pixels in the image, denoted by n. Thus, a normalized histogram is given by $p(r_k) = n_k/n$, for $k = 0, 1, \ldots, L-1$. Loosely speaking, $p(r_k)$ gives an estimate of the probability of occurrence of gray level r_k. Note that the sum of all components of a normalized histogram is equal to 1.

Exemple:

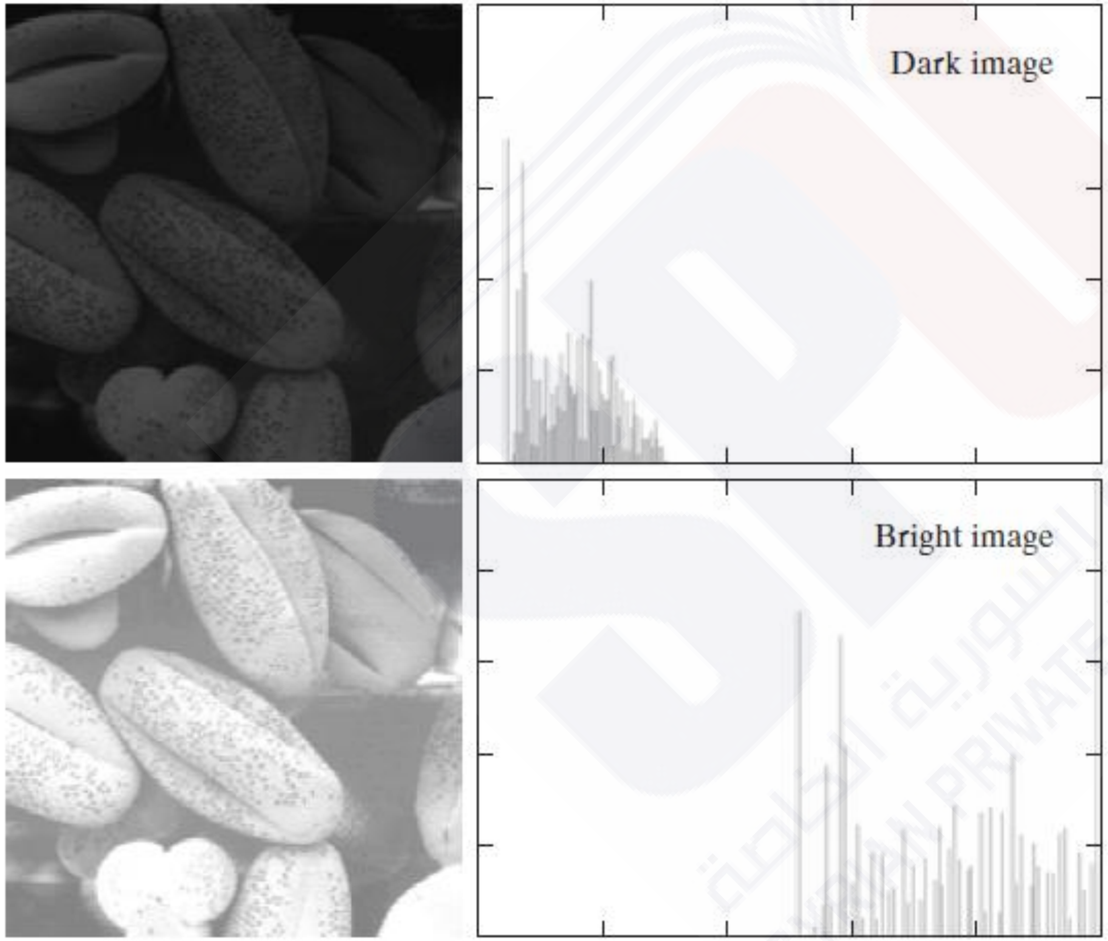
20	22	20	22	15	18	19	20	19	17
20	24	20	22	15	18	19	23	19	17
20	28	20	22	15	18	19	24	19	16
20	12	20	22	15	18	19	20	19	15
20	32	20	22	15	18	19	20	19	17
20	22	20	22	15	20	22	20	22	15
20	22	20	22	15	20	22	20	22	15
20	22	20	22	15	25	22	20	22	25
20	22	20	22	15	22	22	20	23	15
20	22	20	22	15	20	22	20	22	30

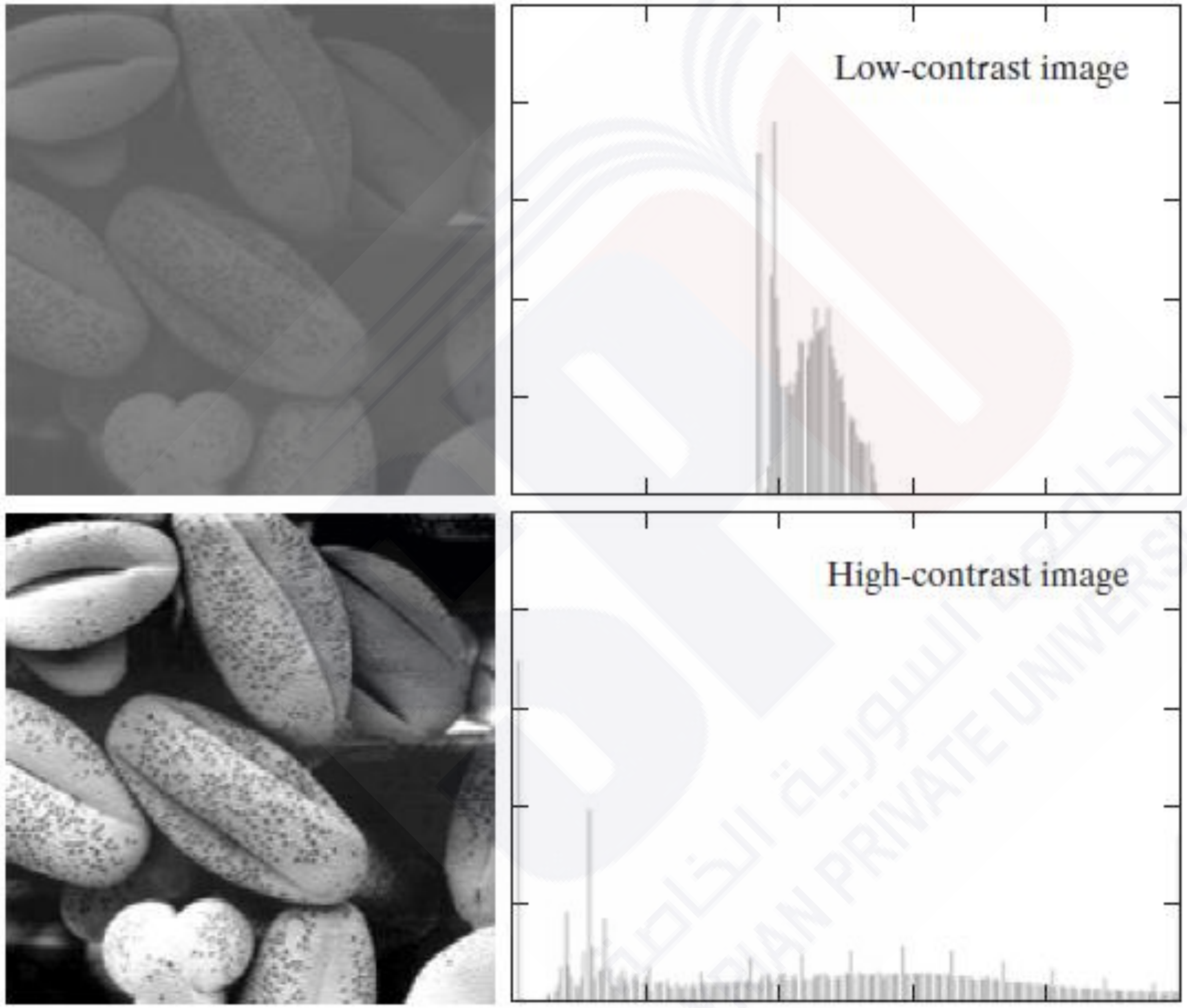


How to calculate a histogram of an image?

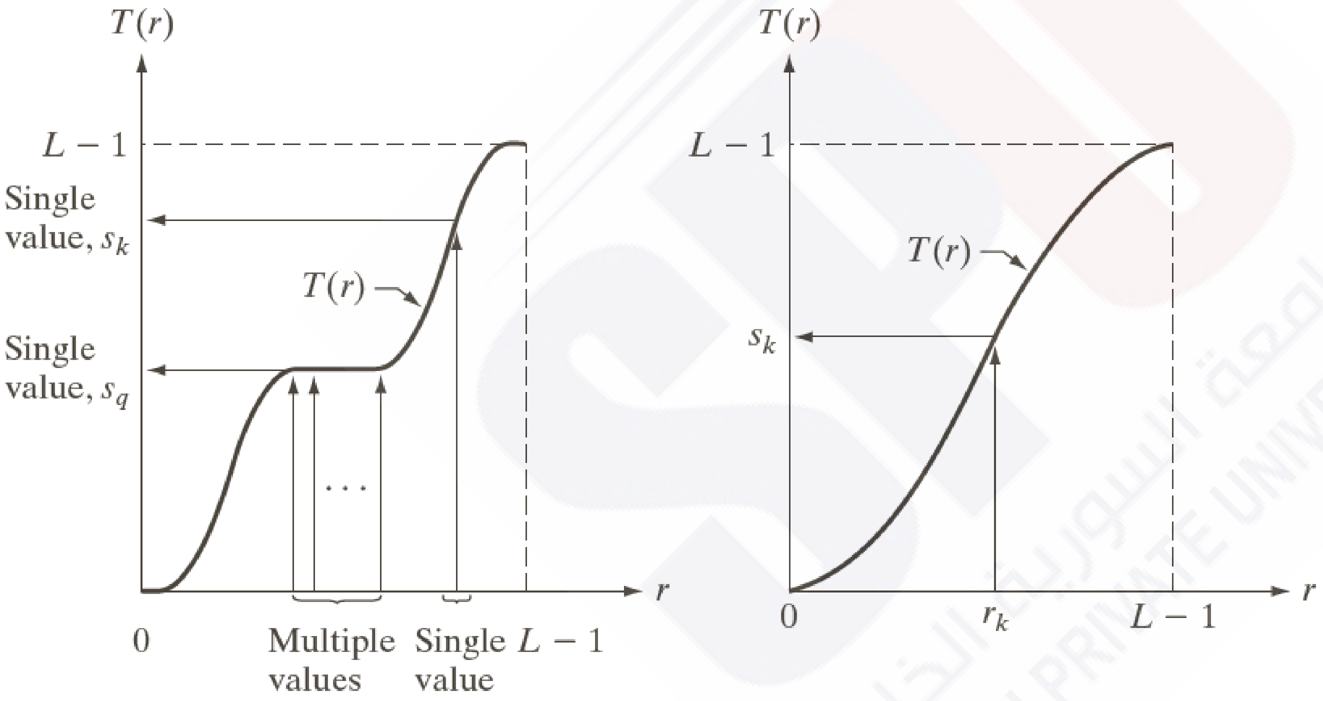
What are the histogram indications to image quality?

Histogram Processing using transformation functions



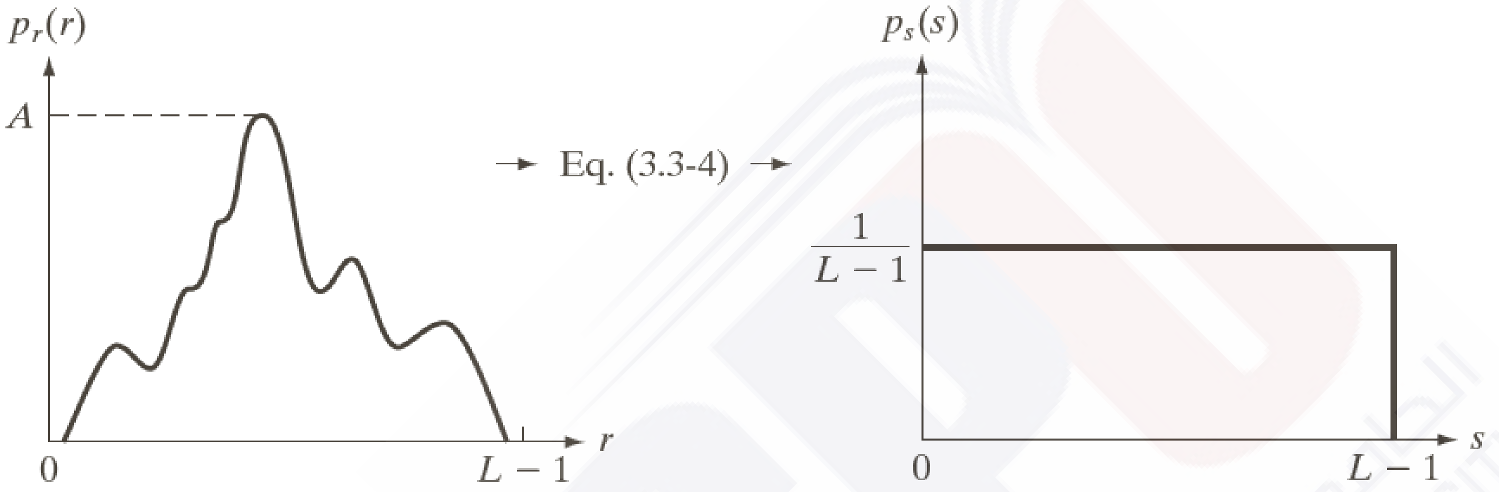

Low-contrast image
High-contrast image

Histogram Equalization



a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.



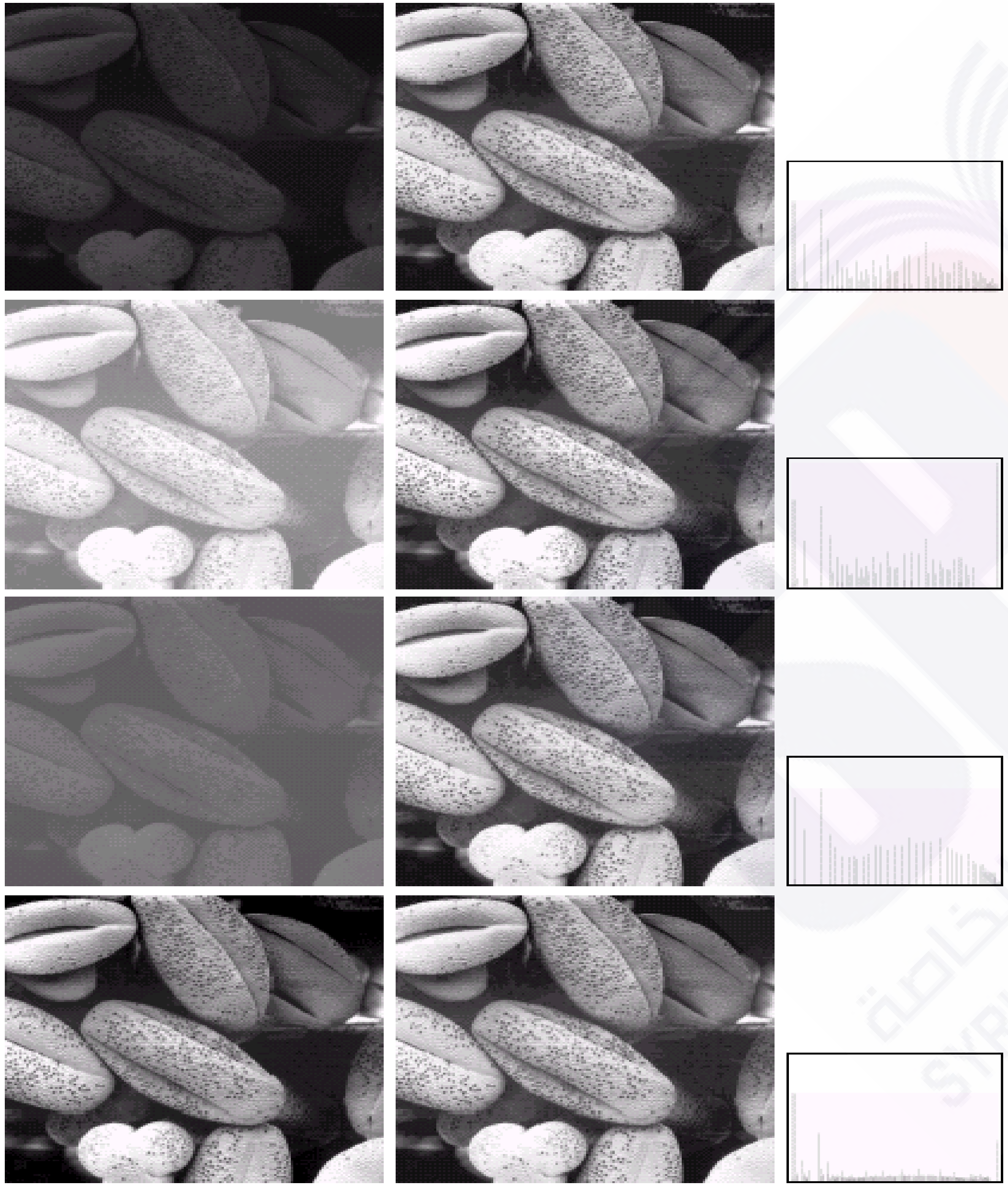
a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

$$s_k = T(r_k) = (L - 1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, 2, \ldots, L - 1$$

Equalization



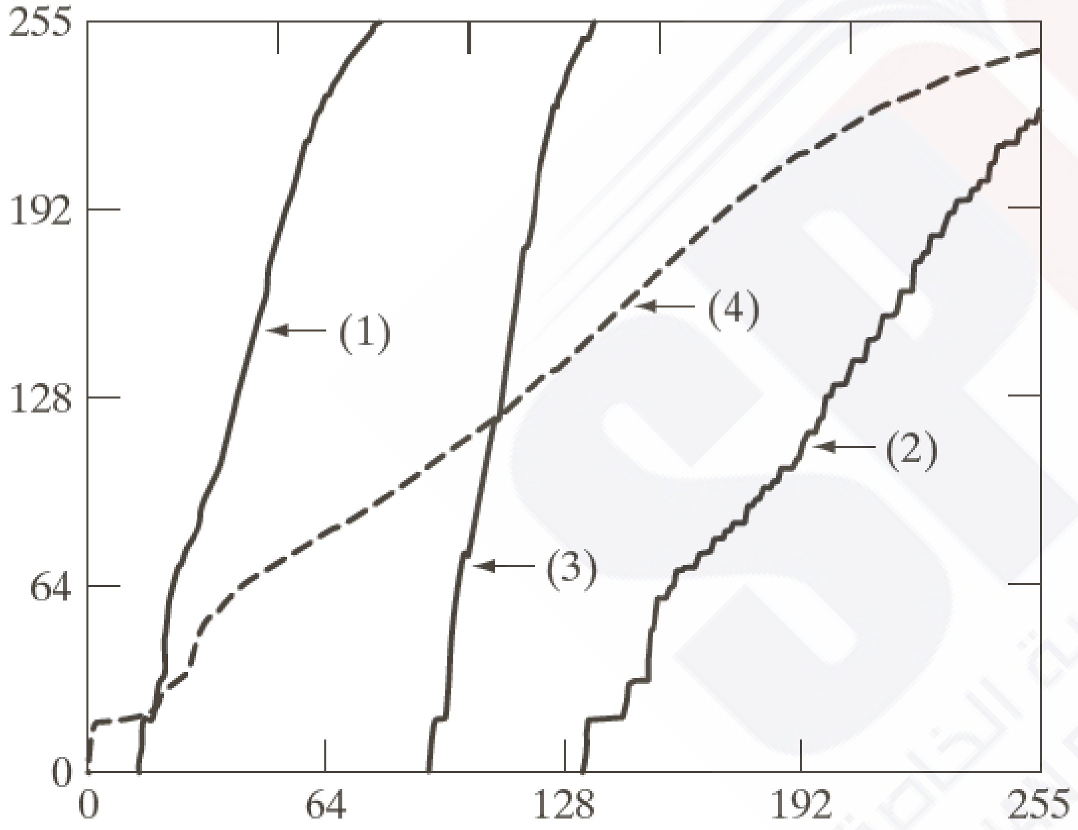


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

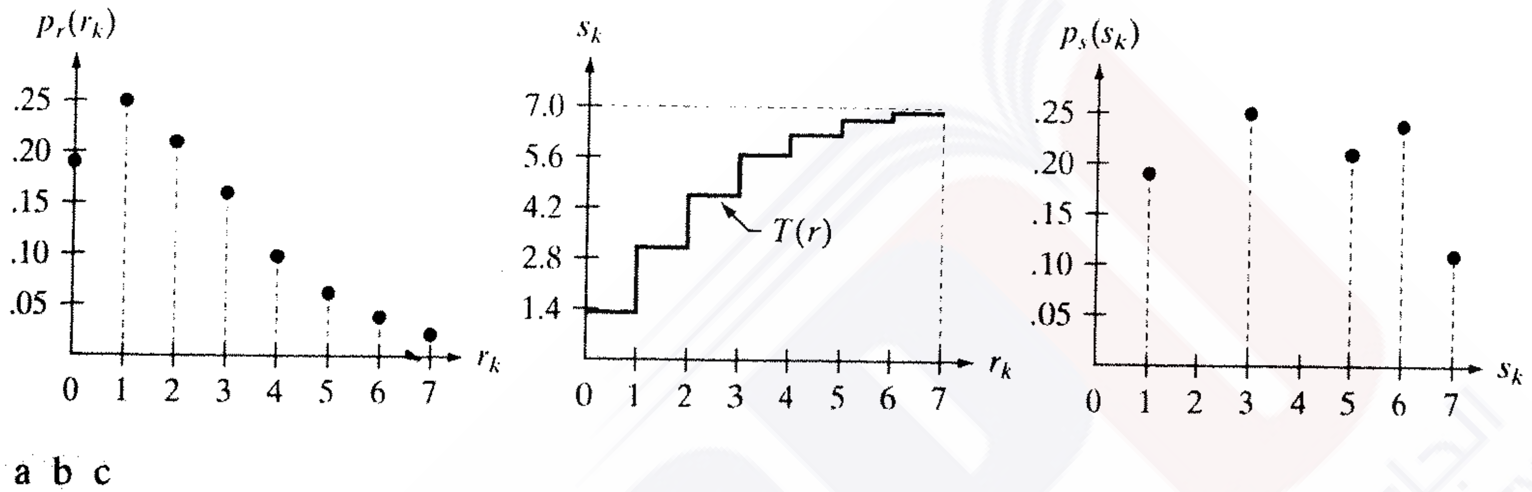
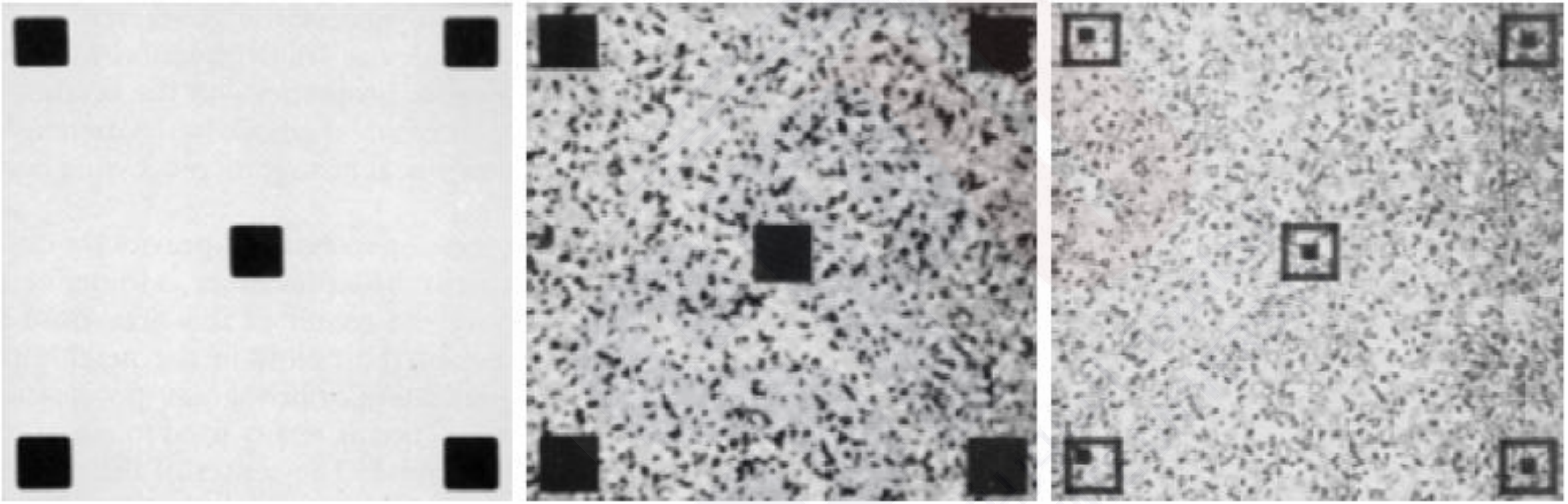


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

At this point, the s values still have fractions because they were generated by summing probability values, so we round them to the nearest integer:

$$s_0 = 1.33 \rightarrow 1 \qquad s_4 = 6.23 \rightarrow 6$$
$$s_1 = 3.08 \rightarrow 3 \qquad s_5 = 6.65 \rightarrow 7$$
$$s_2 = 4.55 \rightarrow 5 \qquad s_6 = 6.86 \rightarrow 7$$
$$s_3 = 5.67 \rightarrow 6 \qquad s_7 = 7.00 \rightarrow 7$$

Local Histogram Processing



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7 × 7 neighborhood about each pixel.

Using Histogram statistics for image Enhancement

To enhance automatically image contrast.

Mean value of histogram (average intensity of image)

$$m = \sum_{i=0}^{L-1} r_i p(r_i) \qquad m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

$$\sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - m]^2$$

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i).$$

END OF PRESENTATION